(b) Express $f(x) = x$ as a half range sine series in $0 < x < 2$. **2½**

Unit III

6. (a) Determine the stereographic projection of the following points on the sphere of radius 1 and centre (0, 0, 0) :

(i) $1 + i$

(ii) $1 - i$. **2½**

(b) Prove that an analytic function with constant modulus is constant. **2½**

7. (a) For what values of z, the function $z = \sinh u \cos v + i \cosh u \sin v$ ceases to be analytic. **2½**

(b) Show that the function $u(x, y) = x^3 - 3xy^2$ is harmonic and find the corresponding analytic function. **2½**

32505

B.A. & Hons. (Subsidiary) EXAMINATION, 2025

(Sixth Semester)

(Regular & Re-appear)

MATHEMATICS

BM-361

Real and Complex Analysis

Time : 3 Hours] [*Maximum Marks :* 27

Before answering the question-paper, candidates must ensure that they have been supplied with correct and complete question-paper. No complaint, in this regard will be entertained after the examination.

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. **1** is compulsory.

(Compulsory Question)

1. (a) If $u = x^2 - x \sin y$ and $v = x^2y^2 + x + y$, then find $\dfrac{\partial(u,v)}{\partial(x,y)}$. **1**

(b) Define Beta function. **1**

(c) State Dirichlet's integral. **1**

(d) Define conformal mapping. **1**

(e) State Parseval's identity for Fourier series. **1**

(f) Show that : $\Gamma\left(\dfrac{1}{2}\right) = \sqrt{\pi}$. **2**

Unit I

2. (a) If $x = r\cos\theta, y = r\sin\theta$, show that : **2½**

$$\frac{\partial(x,y)}{\partial(r,\theta)} \cdot \frac{\partial(r,\theta)}{\partial(x,y)} = 1.$$

(b) Express $\int_0^1 x^m (1-x^n)^p \, dx$ in terms of Beta function and hence evaluate :

$$\int_0^1 x^5 (1-x^3)^3 \, dx.$$ **2½**

3. (a) Prove that $\Gamma(n)\Gamma(1-n) = \dfrac{\Pi}{\sin n\Pi}$, when

$$\int_0^\infty \frac{x^{n-1}}{1+x} dx = \frac{\Pi}{\sin n\Pi}.$$ **2½**

(b) Evaluate the integral $\int_0^1 \int_0^x \int_0^{1+x+y} f(x,y,z) dx\, dy\, dz$ where $f(x, y, z) = 1$. **2½**

Unit II

4. (a) Find the Fourier expansion of $f(x) = x$ in $[-\pi, \pi]$. **2½**

(b) Obtain the Fourier series expansion of the function f in $(0, 2\pi)$ defined as :

$$f(x) = \begin{cases} x \text{ for } 0 < x < \pi \\ 0 \text{ for } \pi < x < 2\pi \end{cases}.$$ **2½**

5. (a) Find the Fourier series expansion of the function f with period 2π defined as :

$$f(x) = \begin{cases} -1 \text{ for } -\pi < x < 0 \\ \ \ 1 \text{ for } 0 \le x \le \pi \end{cases}.$$ **2½**

Unit IV

8. (a) Find the fixed points, normal form and nature of bilinear transformation $w = \frac{z}{z-2}$. **2½**

(b) Find the Mobius transformation which maps the points $z_1 = 2$, $z_2 = i$, $z_3 = -2$, into the points $w_1 = 1$, $w_2 = i$, $w_3 = -1$. **2½**

9. (a) Find the bilinear transformation which maps the points $z = 0, -1, i$ onto $w = i, 0, \infty$, Also find the image of the unit circle $|z| = 1$. **2½**

(b) Show that the family of circles $|w - 1| = k$ is transformed into the family of lemniscate $|z - 1||z + 1| = k$ under the transformation $w = z^2$. **2½**

❖❖❖❖❖❖❖

Unit IV

8. (a) Find the fixed points, normal form and nature of bilinear transformation $w = \frac{z}{z-2}$. **2½**

(b) Find the Mobius transformation which maps the points $z_1 = 2$, $z_2 = i$, $z_3 = -2$, into the points $w_1 = 1$, $w_2 = i$, $w_3 = -1$. **2½**

9. (a) Find the bilinear transformation which maps the points $z = 0, -1, i$ onto $w = i, 0, \infty$, Also find the image of the unit circle $|z| = 1$. **2½**

(b) Show that the family of circles $|w - 1| = k$ is transformed into the family of lemniscate $|z - 1||z + 1| = k$ under the transformation $w = z^2$. **2½**

❖❖❖❖❖❖❖